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ij-$\ddot{g}$-continuous map in Bitopological Spaces

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Abstract- The aim of this paper is to introduce ij-$\ddot{g}$-continuous map, ij-$\ddot{g}$-irresolute map in Bitopological Spaces and to study about their properties.

Keywords: $\boldsymbol{ij}$-$\boldsymbol{\ddot{g}}$-continuous map; $\boldsymbol{ij}$-$\boldsymbol{\ddot{g}}$-irresolute map.

1. INTRODUCTION

Kelly [7] initiated the study of bitopological spaces in 1963. A nonempty set X equipped with two topologies τ_1 and τ_2 is called a bitopological space and is denoted by (X, τ_1, τ_2). Since then several topologists generalized many of the results in topological spaces to bitopological spaces. Fukutake [3] introduced generalized closed sets in bitopological spaces. Fukutake [4] defined semi open sets in bitopological spaces. In 2012, Qays H. I. Al-Rubaye[9] introduced Semi- α-pen, semi -α- separation in Bitopological space.

2. PRELIMINARIES

Definition 2.1: A subset A of a bitopological space (X, τ_1, τ_2) is called a

1. $\tau_1\tau_2$-semi open [1] if $A \subset \tau_2 - cl(\tau_1 - int(A))$ and it is called $\tau_1\tau_2$-semi closed [2] if $\tau_1 - int(\tau_2 - cl(A)) \subset A$
2. $\tau_1\tau_2$-pre open[6] if $A \subset \tau_1 - int(\tau_2 - cl(A))$ and $\tau_1\tau_2$-pre closed [5] if $\tau_2 - cl(\tau_1 - int(A)) \subset A$
3. $\tau_1\tau_2$-α-open [10] if $A \subset \tau_1 - int\left(\tau_2 - cl(\tau_1 - int(A))\right)$.

Definition 2.2: A subset A of a bitopological space (X, τ_1, τ_2) is called a

1. $\tau_1\tau_2$-g-closed [3]($\tau_1\tau_2$-generalized closed) if τ_2-$cl(A) \subset U$, whenever $A \subset U$, U is τ_1-open.
2. $\tau_1\tau_2$-sg-closed [4]($\tau_1\tau_2$-semi generalized closed) if τ_2-$scl(A) \subset U$, whenever $A \subset U$, U is τ_1-semi open.
3. $\tau_1\tau_2$-gs-closed [6] ($\tau_1\tau_2$-generalized semi closed) if τ_2-$scl(A) \subset U$, whenever $A \subset U$, U is τ_1-open.
4. $\tau_1\tau_2$-αg-closed [10] ($\tau_1\tau_2$-α-generalized closed) if τ_2-$\alpha cl(A) \subset U$, whenever $A \subset U$, U is τ_1-open.
5. $\tau_1\tau_2$-gα-closed [11] ($\tau_1\tau_2$-generalized α-closed) if τ_2-$\alpha cl(A) \subset U$, whenever $A \subset U$, U is τ_1-α-open.
6. ij-$\hat{g}$-closed[6] if τ_2-$cl(A) \subset U$, whenever $A \subset U$, U is τ_1- semi open.
7. $\tau_1\tau_2$-semi-α-closed[8] if τ_2-$cl(A) \subset U$, whenever $A \subset U$, U is $\tau_1\tau_2$- semi α- open.

Definition 2.3 [5]:
A subset A of (X, τ_1, τ_2) is called a $\tau_1\tau_2$-$\ddot{g}$-closed(resp. $\tau_2\tau_1$-$\ddot{g}$-closed) if τ_2-$cl(A) \subset U$,(rep. τ_1-$cl(A) \subset U$) whenever $A \subset U$ and U is τ_1-sg-open (resp. τ_2-sg-open).

Results 2.4:[5]

1. Every τ_2-closed set is $\tau_1\tau_2$-$\ddot{g}$-closed sets.
2. Every $\tau_1\tau_2$-$\ddot{g}$-closed set is $\tau_1\tau_2$-g-closed.
3. Every $\tau_1\tau_2$-$\ddot{g}$-closed set is $\tau_1\tau_2$-$\hat{g}$- closed.
4. Every $\tau_1\tau_2$-$\ddot{g}$-closed set is $\tau_1\tau_2$-gs-closed.
5. Every $\tau_1\tau_2$-$\ddot{g}$-closed set is $\tau_1\tau_2$-αg-closed.
6. Every $\tau_1\tau_2$-$\ddot{g}$-closed set is $\tau_1\tau_2$-gα-closed.
7. Arbitrary union of $\tau_1\tau_2$-$\ddot{g}$-closed sets $\{A_i, i \in I\}$ in a bitopological space (X, τ_1, τ_2) is $\tau_1\tau_2$-$\ddot{g}$-closed if the family $\{A_i, i \in I\}$ is locally finite on X.

Definition 2.5 [9]:
A function $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is said to be ij-semi-α-continuous if the inverse image of each i-open set of Y is ij-semi-α-open in X, where $i \neq j$ and $i, j = 1,2$.

Definition 2.6[2]:
A function $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is said to be pairwise continuous iff the induced functions are $f: (X, \tau_1) \rightarrow (Y, \sigma_1)$ and $f: (X, \tau_2) \rightarrow (Y, \sigma_2)$ are continuous.

3. ij-$\ddot{g}$-CONTINUOUS

Definition 3.1: A function $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ is said to be ij-$\ddot{g}$-continuous if the inverse image of each σ_j-closed set of Y is $\tau_i\tau_j$-$\ddot{g}$-closed in X, where $i\neq j$ and $i,j=1,2$.

Theorem 3.2: *Every pairwise continuous function is ij-$\ddot{g}$-continuous but not conversely.*
Proof: Let $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ be pairwise continuous. Let V be σ_j-closed set in Y. Then $f^{-1}(V)$ is τ_j-closed set in X, where $j=1,2$. Since every τ_j-closed set is $\tau_i\tau_j$-$\ddot{g}$-closed set, where $i\neq j$ and $i,j=1,2$. Therefore $f^{-1}(V)$ is $\tau_i\tau_j$-$\ddot{g}$-closed set in X, where $i\neq j$ and $i,j=1,2$. Hence f is ij-$\ddot{g}$-continuous.

Example 3.3:
Let $X=\{a,b,c,d\},\tau_1=\{X,\emptyset,\{b\},\{b,d\},\{b,c,d\}\}$, $\tau_2=\{X,\emptyset,\{a\},\{a,d\},\{a,c,d\}\}$,
$\sigma_1=\{Y,\emptyset,\{a\},\{a,b\}\}$ and $\sigma_2=\{Y,\emptyset,\{b\},\{a,b\}\}$.
Define a function $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ be an identity function. Now, let $\{b,c,d\}$ is σ_1-closed set in Y. Then $f^{-1}(\{b,c,d\})=\{b,c,d\}$ is $\tau_2\tau_1$-$\ddot{g}$-closed set but not τ_1-closed set.
Again, let $\{c,d\}$ is σ_2-closed set in Y. Then $f^{-1}(\{c,d\})=\{c,d\}$ is $\tau_1\tau_2$-$\ddot{g}$-closed set but not τ_2-closed set. Hence f is ij-$\ddot{g}$-continuous but not pairwise continuous.

Theorem 3.4: *The ij-$\ddot{g}$-continuous function $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ is independent of $\ddot{g}$-continuousness of the induced functions $f:(X,\tau_1)\to(Y,\sigma_1)$ and $f:(X,\tau_2)\to(Y,\sigma_2)$ as can be seen in the following examples*

Example 3.5:
Let $X=Y=\{a,b,c,d\}$,
$\tau_1=\{X,\emptyset,\{a\},\{b,c\},\{a,b,c\}\}$,
$\tau_2=\{X,\emptyset,\{a\},\{b,d\},\{a,c\},\{a,b,d\}\}$,
$\sigma_1=\{Y,\emptyset,\{a,b\}\}$ and $\sigma_2=\{Y,\emptyset,\{d\},\{a,d\}\}$.
Define a function $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ be an identity function. Now, let $\{c,d\}$ is σ_1-closed set in Y. Then $f^{-1}(\{c,d\})=\{c,d\}$ is τ_1-$\ddot{g}$-closed set but not $\tau_2\tau_1$-$\ddot{g}$-closed set. Again, let $\{b,c\}$ is σ_2-closed set in Y. Then $f^{-1}(\{b,c\})=\{b,c\}$ is τ_2-$\ddot{g}$-closed set but not $\tau_1\tau_2$-$\ddot{g}$ closed set. Hence the induced function is $\ddot{g}$-continous but not ij-$\ddot{g}$-continuous.

Example 3.6:
Let $X=Y=\{a,b,c,d\}$
$\tau_1=\{X,\emptyset,\{b\},\{b,d\},\{b,c,d\}\}$,
$\tau_2=\{X,\emptyset,\{a\},\{a,c\}\},\sigma_1=\{Y,\emptyset,\{a\},\{a,b\}\}$
and $\sigma_2=\{Y,\emptyset,\{a,b\},\{a,b,c\}\}$.
Define a function $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ be an identity function. Now, let $\{c,d\}$ is σ_1-closed set in Y. Then $f^{-1}(\{c,d\})=\{c,d\}$ is $\tau_2\tau_1$-$\ddot{g}$-closed set but not τ_1-closed set. Hence f is ij-$\ddot{g}$-continuous but not the induced function is $\ddot{g}$-continuous.

Theorem 3.7: *Every ij-$\ddot{g}$-continuous is ij-g-continuous but not conversely.*
Proof: Let $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ be ij-$\ddot{g}$-continuous. Let V be σ_j-closed set in Y, where $j=1,2$. Then $f^{-1}(V)$ is $\tau_i\tau_j$-$\ddot{g}$-closed set in X, where $i\neq j$ and $i,j=1,2$. Since every $\tau_i\tau_j$-$\ddot{g}$-closed set is $\tau_i\tau_j$-g-closed. Therefore $f^{-1}(V)$ is $\tau_i\tau_j$-g-closed set.

Example 3.8:
Let $X=Y=\{a,b,c\},\tau_1=\{X,\emptyset,\{c\},\{a,b\}\}$,
$\tau_2=\{X,\emptyset,\{a\},\{b\},\{c\},\{a,b\}\},\sigma_1=\{Y,\emptyset,\{b,c\}\}$ and $\sigma_2=\{Y,\emptyset,\{a,c\}\}$.
Define a mapping $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ by $f(a)=a;f(b)=b;f(c)=c$. Hence f is ij-g-continuous but not ij-$\ddot{g}$-continuous. Since $\{a\},\{b\}$ are $\tau_1\tau_2$-g-closed set but not $\tau_1\tau_2$-$\ddot{g}$-closed set.

Theorem 3.9: *Every ij-$\ddot{g}$-continuous is ij-$\hat{g}$-continuous but not conversely.*
Proof: Let $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ be ij-$\ddot{g}$-continuous. Let V be σ_j-closed set in Y, where $j=1,2$. Then $f^{-1}(V)$ is $\tau_i\tau_j$-$\ddot{g}$-closed set in X, where $i\neq j$ and $i,j=1,2$. Since every $\tau_i\tau_j$-$\ddot{g}$-closed set is $\tau_i\tau_j$-$\hat{g}$-closed. Therefore $f^{-1}(V)$ is $\tau_i\tau_j$-$\hat{g}$-closed set.

Example 3.10:
Let $X=Y=\{a,b,c\},\tau_1=\{X,\emptyset,\{c\},\{a,b\}\}$,
$\tau_2=\{X,\emptyset,\{a\},\{b\},\{c\},\{a,b\}\},\sigma_1=\{Y,\emptyset,\{b,c\}\}$ and $\sigma_2=\{Y,\emptyset,\{a,c\}\}$.
Define a mapping $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ by $f(a)=a;f(b)=b;f(c)=c$. Hence f is ij-$\hat{g}$-continuous but not ij-$\ddot{g}$-continuous. Since $\{a\},\{b\}$ are $\tau_1\tau_2$-$\hat{g}$-closed set but not $\tau_1\tau_2$-$\ddot{g}$-closed set.

Theorem 3.11: *Every ij-$\ddot{g}$-continuous is ij-gs-continuous but not conversely.*
Proof: Let $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ be ij $\ddot{g}$-continuous. Let V be σ_j-closed set in Y, where $j=1,2$. Then $f^{-1}(V)$ is $\tau_i\tau_j$-$\ddot{g}$-closed set in X, where $i\neq j$ and $i,j=1,2$. Since every $\tau_i\tau_j$-$\ddot{g}$-closed set is $\tau_i\tau_j$-gs-closed. Therefore $f^{-1}(V)$ is $\tau_i\tau_j$-gs-closed set.

Example 3.12:
Let $X=Y=\{a,b,c\},\tau_1=\{X,\emptyset,\{a\},\{b\},\{a,b\}\}$,
$\tau_2=\{X,\emptyset,\{a\}\},\sigma_1=\{Y,\emptyset,\{b\}\}$ and $\sigma_2=\{Y,\emptyset,\{c\}\}$.
Define a mapping $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ by $f(a)=a;f(b)=b;f(c)=c$. Hence f is ij-gs-continuous but not ij -$\ddot{g}$-continuous. Since $\{b\},\{c\}$ are $\tau_1\tau_2$-gs-closed set but not $\tau_1\tau_2$-$\ddot{g}$-closed set.

Theorem 3.13: *Every ij-$\ddot{g}$-continuous is ij-αg-continuous but not conversely.*

Proof: Let $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ be ij $\ddot{g}$-continuous. Let V be σ_j-closed set in Y, where $j=1,2$. Then $f^{-1}(V)$ is $\tau_i\tau_j$-$\ddot{g}$-closed set in X, where $i\neq j$ and $i,j=1,2$. Since every $\tau_i\tau_j$-$\ddot{g}$-closed set is $\tau_i\tau_j$-αg-closed. Therefore $f^{-1}(V)$ is $\tau_i\tau_j$-αg-closed set.

Example 3.14:
Let $X=Y=\{a,b,c\}, \tau_1=\{X,\emptyset,\{b\},\{c\},\{b,c\}\}$, $\tau_2=\{X,\emptyset,\{a\},\{b\},\{a,b\}\}, \sigma_1=\{Y,\emptyset,\{c\},\{a,b\}\}$ and $\sigma_2=\{Y,\emptyset,\{a,c\}\}$.Define a mapping $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ by $f(a)=a; f(b)=b; f(c)=c$. Hence f is ij-αg-continuous but not ij-$\ddot{g}$-continuous. Since $\{b\},\{c\}$ are $\tau_1\tau_2$-$\hat{g}$-closed set but not $\tau_1\tau_2$-$\ddot{g}$-closed set.

Theorem 3.15: *Every ij-$\ddot{g}$-continuous is ij-$g\alpha$-continuous but not conversely.*

Proof: Let $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ be ij $\ddot{g}$-continuous. Let V be σ_j-closed set in Y, where $j=1,2$. Then $f^{-1}(V)$ is $\tau_i\tau_j$-$\ddot{g}$-closed set in X, where $i\neq j$ and $i,j=1,2$. Since every $\tau_i\tau_j$-$\ddot{g}$-closed set is $\tau_i\tau_j$-$g\alpha$-closed. Therefore $f^{-1}(V)$ is $\tau_i\tau_j$-$g\alpha$-closed set.

Example 3.16:
Let $X=Y=\{a,b,c\}, \tau_1=\{X,\emptyset,\{a\},\{b\},\{a,b\}\}$, $\tau_2=\{X,\emptyset,\{b\},\{c\},\{b,c\}\}, \sigma_1=\{Y,\emptyset,\{a,c\}\}$ and $\sigma_2=\{Y,\emptyset,\{a\},\{c\},\{a,c\}\}$.
Define a mapping $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ by $f(a)=a; f(b)=b; f(c)=c$. Hence f is ij-$g\alpha$-continuous but not ij-$\ddot{g}$-continuous. Since $\{b\}$ are $\tau_1\tau_2$-$\hat{g}$-closed set but not $\tau_1\tau_2$-$\ddot{g}$-closed set.

Theorem 3.17: *If $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ is ij-$\ddot{g}$-continuous and $g:(Y,\sigma_1,\sigma_2)\to(Z,\eta_1,\eta_2)$ is pairwise continuous then $g\circ f$ is ij-$\ddot{g}$-continuous.*

Proof: Let η_i-closed set in Z. Since g is pairwise continuous, then $g^{-1}(V)$ is σ_j-closed set in Y, where $j=1,2$. Again f is ij-$\ddot{g}$-continuous, $(g\circ f)^{-1}(V)=f^{-1}\big(g^{-1}(V)\big)$ is $\tau_i\tau_j$-$\ddot{g}$-closed set in X, where $i\neq j$ and $i,j=1,2$.Which implies that $(g\circ f)^{-1}(V)$ is $\tau_i\tau_j$-$\ddot{g}$-closed set in X, where $i\neq j$ and $i,j=1,2$. Hence $g\circ f$ is ij-$\ddot{g}$-continuous.

Theorem 3.18: *For a function $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ the following conditions are equivalent.*

(i) *f is ij-$\ddot{g}$-continuous.*
(ii) *$f^{-1}(U)$ is $\tau_i\tau_j$-$\ddot{g}$-open for each σ_j-open set U in Y, where $i\neq j$ and $i,j=1,2$.*

Proof: $(a)\Rightarrow(b)$ Suppose that f is ij $\ddot{g}$-continuous. Let V be σ_j-open in Y. Then V^c is σ_j-closed in Y, $j=1,2$. Since f is ij-$\ddot{g}$-continuous, we have $f^{-1}(V^c)$ is $\tau_i\tau_j$-$\ddot{g}$-closed set in X, where $i\neq j$ and $i,j=1,2$. Consequently, $f^{-1}(V)$ is $\tau_i\tau_j$-$\ddot{g}$-open set in X, where $i\neq j$ and $i,j=1,2$.
$(b)\Rightarrow(a)$ Suppose that $f^{-1}(U)$ is $\tau_i\tau_j$-$\ddot{g}$-open set for each σ_j-open set U in Y, where $i\neq j$ and $i,j=1,2$. Let V be σ_j-closed in Y. Then V^c be σ_j-open in Y. By our assumption, $f^{-1}(V^c)$ is $\tau_i\tau_j$-$\ddot{g}$-open in X, where $i\neq j$ and $i,j=1,2$. Hence $f^{-1}(V)$ is $\tau_i\tau_j$-$\ddot{g}$-closed set in X, where $i\neq j$ and $i,j=1,2$. Therefore f is ij-$\ddot{g}$-continuous.

Definition 3.19: A function $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ is said to be ij-$\ddot{g}$-irresolute if $f^{-1}(V)$ is $\tau_i\tau_j$-$\ddot{g}$-closed set in X for each $\sigma_i\sigma_j$-$\ddot{g}$-closed set V in Y, where $i\neq j$ and $i,j=1,2$.

Theorem 3.20: *Let $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ and $g:(Y,\sigma_1,\sigma_2)\to(Z,\eta_1,\eta_2)$ be two functions. Then*

(1) *$g\circ f$ is ij-$\ddot{g}$-continuous if g is ij-$\ddot{g}$-continuous and f is ij-$\ddot{g}$-irresolute.*
(2) *$g\circ f$ is ij-$\ddot{g}$-irresolute if both f and g are ij-$\ddot{g}$-irresolute.*

Proof:

1. Let V be η_j-closed in Z. Since g is ij-$\ddot{g}$-continuous, $g^{-1}(V)$ is $\sigma_i\sigma_j$-$\ddot{g}$-closed in Y, where $i\neq j$ and $i,j=1,2$. Since f is ij-$\ddot{g}$-irresolute, $f^{-1}\big(g^{-1}(V)\big)=(g\circ f)^{-1}(V)$ is $\tau_i\tau_j$-$\ddot{g}$-closed in X, where $i\neq j$ and $i,j=1,2$. Which implies that $(g\circ f)^{-1}(V)$ is $\tau_i\tau_j$-$\ddot{g}$-closed in X, where $i\neq j$ and $i,j=1,2$. Hence $g\circ f$ is ij-$\ddot{g}$-continuous.
2. Let V be $\eta_i\eta_j$-$\ddot{g}$-closed in Z, where $i\neq j$ and $i,j=1,2$. Since g is ij-$\ddot{g}$-irresolute, $g^{-1}(V)$ is $\sigma_i\sigma_j$-$\ddot{g}$-closed set in Y, where $i\neq j$ and $i,j=1,2$. As f is ij-$\ddot{g}$-irresolute, $f^{-1}\big(g^{-1}(V)\big)=(g\circ f)^{-1}(V)$ is $\tau_i\tau_j$-$\ddot{g}$-closed in X, where $i\neq j$ and $i,j=1,2$. which implies that $(g\circ f)^{-1}(V)$ is $\tau_i\tau_j$-$\ddot{g}$-closed in X, where $i\neq j$ and $i,j=1,2$. Hence $g\circ f$ is ij-$\ddot{g}$-irresolute.

Theorem 3.21: *If a function $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ is ij-$\ddot{g}$-irresolute, then it is ij-$\ddot{g}$-continuous.*

Proof: Let $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ be ij-$\ddot{g}$-irresolute. Let V be any σ_j-closed set in Y, $j=1,2$. Since every σ_j-closed set is $\sigma_i\sigma_j$-$\ddot{g}$-closed set, then V is $\sigma_i\sigma_j$-$\ddot{g}$-closed set in Y, where $i\neq j$ and $i,j=1,2$. Which implies that $f^{-1}(V)$ is $\tau_i\tau_j$-$\ddot{g}$-closed set in X, where $i\neq j$ and $i,j=1,2$. Hence f is ij-$\ddot{g}$-continuous.

Theorem 3.22: *Let $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ be a function, then the following statements are equivalent.*

(1) *f is ij-$\ddot{g}$-irresolute function.*

(2) *For $x \in X$ and each $\sigma_i\sigma_j$-$\ddot{g}$-closed set V of Y containing $f(x)$, there exists and $\tau_i\tau_j$-$\ddot{g}$-closed set U such that $x \in U$ and $f(U) \subset V$, where $i \neq j$ and $i, j = 1{,}2$.*

(3) *The inverse image of every $\sigma_i\sigma_j$-$\ddot{g}$-open set of Y is $\tau_i\tau_j$-$\ddot{g}$-open in X, where $i \neq j$ and $i, j = 1{,}2$.*

Proof:

$(1) \Rightarrow (2)$ Let V be an $\sigma_i\sigma_j$-$\ddot{g}$-closed set of Y, where $i \neq j$ and $i, j = 1{,}2$ and $f(x) \in V$. Since f is ij-$\ddot{g}$-irresolute, $f^{-1}(V)$ is $\tau_i\tau_j$-$\ddot{g}$-closed set in X, where $i \neq j$ and $i, j = 1{,}2$ and $x \in f^{-1}(V)$. Put $U = f^{-1}(V)$. Then $x \in U$ and $f(U) \subset V$.

$(2) \Rightarrow (1)$ Let V be an $\sigma_i\sigma_j$-$\ddot{g}$-closed set of Y, where $i \neq j$ and $i, j = 1{,}2$ and $x \in f^{-1}(V)$. Then $f(x) \in V$. Therefore by (2), there exists an $\tau_i\tau_j$-$\ddot{g}$-closed set (where $i \neq j$ and $i, j = 1{,}2$) U_x such that $x \in U_x$ and $f(U_x) \subset V$. Hence $x \in U_x \subset f^{-1}(V)$. Which implies that $f^{-1}(V)$ is a union of $\tau_i\tau_j$-$\ddot{g}$-closed set of X, where $i \neq j$ and $i, j = 1{,}2$(By known result 2.4-7)Thus $f^{-1}(V)$ is $\tau_i\tau_j$-$\ddot{g}$-closed set, where $i \neq j$ and $i, j = 1{,}2$. Hence f is ij-$\ddot{g}$-irresolute function.

$(1) \Leftrightarrow (3)$ The proof is obvious.

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